COMPLETING THE SQUARE

lesson link: parkermaths.com/y1compsquare

Objective

- To write quadratic expressions in completed square form: $a(x+b)^2+c$

Examples

3.1e. Express the following in the form $(x+a)^2+b$, where a and b are rational numbers.

(a) x^2-6x+4

$$x^2-6x+4=(x-3)^2-9+4$$
$$=(x-3)^2-5$$

(b) y^2+5y-6

$$y^2+5y-6=\left(y+\frac{5}{2}\right)^2-\frac{25}{4}-\frac{24}{4}$$
$$=\left(y+\frac{5}{2}\right)^2-\frac{49}{4}$$

TOP TIP

Write both parts as fractions with a common denominator.

3.1p. Express the following in the form $(x+a)^2+b$, where a and b are rational numbers.

(a) $x^2+4x+11$

(b) y^2-3y+8

3.2e. Write $2x^2+7x+9$ in the form $a(x+b)^2+c$, where a,b and c are rational numbers.

Factorise the first two terms.

$$2x^2+7x+9=2\left[x^2+\frac{7}{2}x\right]+9$$

Complete the square

$$=2\left[\left(x+\frac{7}{4}\right)^2-\frac{49}{16}\right]+9$$

Expand

$$=2\left(x+\frac{7}{4}\right)^2-\frac{49}{8}+\frac{72}{8}$$
$$=2\left(x+\frac{7}{4}\right)^2+\frac{23}{8}$$

3.2p. Write $3x^2+2x-5$ in the form $p(x+q)^2+r$, where p,q and r are rational numbers.

3.3e. Write $6-3x-x^2$ in completed square form.

$6-3x-x^2 = -x^2-3x+6$ ← Rewrite in descending powers of x (optional)

$= -\left[x^2+3x\right]+6$ ← Factorise to get a positive x^2 term.

$= -\left[\left(x+\frac{3}{2}\right)^2-\frac{9}{4}\right]+6$ ← Complete the square

$= -\left(x+\frac{3}{2}\right)^2+\frac{9}{4}+\frac{24}{4}$ ← Expand and simplify

$= \frac{33}{4}-\left(x+\frac{3}{2}\right)^2$

3.3p. Write $9+5y-y^2$ in completed square form.

3.4e. Given that $x^2+10px-7\equiv(x+a)^2+b$, express a and b in terms of p.

$x^2+10px-7 = \left(x+5p\right)^2-25p^2-7$

$\therefore\ a = 5p$ and $b = -25p^2-7$

3.4p. Given that $x^2+6qx+3\equiv(x+c)^2+d$, express c and d in terms of q.