General Certificate of Education
June 2005
Advanced Level Examination

MATHEMATICS **MPC4**
Unit Pure Core 4

Friday 24 June 2005 Morning Session

In addition to this paper you will require:
- an 8-page answer book;
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC4.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) Express $2\sin x + \cos x$ in the form $R\sin(x + \alpha)$ where R is a positive constant and α is an acute angle. Give your value of α to the nearest $0.1°$. *(3 marks)*

(b) Solve the equation $2\sin x + \cos x = 1$ for $0° \leqslant x < 360°$. *(4 marks)*

2 (a) Express $\dfrac{3x-5}{(x+3)(2x-1)}$ in the form $\dfrac{A}{x+3} + \dfrac{B}{2x-1}$. *(3 marks)*

(b) Hence find $\displaystyle\int \frac{3x-5}{(x+3)(2x-1)}\,\mathrm{d}x$. *(3 marks)*

3 (a) Find the remainder when $2x^3 - x^2 + 2x - 2$ is divided by $2x - 1$. *(2 marks)*

(b) Given that $\dfrac{2x^3 - x^2 + 2x - 2}{2x-1} = x^2 + a + \dfrac{b}{2x-1}$, find the values of a and b. *(4 marks)*

4 (a) Find the binomial expansion of $(1+x)^{-\frac{1}{2}}$ up to the term in x^2. *(2 marks)*

(b) Hence, or otherwise, obtain the binomial expansion of $\dfrac{1}{\sqrt{1+2x}}$ up to the term in x^2, in simplified form. *(3 marks)*

(c) Use your answer to part (b) with $x = -0.1$ to show that $\sqrt{5} \approx 2.23$. *(3 marks)*

5 A curve is defined by the parametric equations

$$x = 2t + \frac{1}{t}\ , \quad y = \frac{1}{t}\ , \quad t \neq 0$$

(a) Find the coordinates of the point on the curve where $t = \frac{1}{2}$. *(2 marks)*

(b) Show that the cartesian equation of the curve can be written as

$$xy - y^2 = 2$$

(2 marks)

(c) Show that the gradient of the curve at the point $(3, 2)$ is 2. *(6 marks)*

6 (a) Express $\sin 2x$ in terms of $\sin x$ and $\cos x$. *(1 mark)*

(b) Using the identity $\cos(A+B) = \cos A \cos B - \sin A \sin B$:

(i) express $\cos 2x$ in terms of $\sin x$ and $\cos x$; *(2 marks)*

(ii) show, by writing $3x$ as $(2x + x)$, that

$$\cos 3x = 4\cos^3 x - 3\cos x$$ *(4 marks)*

(c) Show that $\int_0^{\frac{\pi}{2}} \cos^3 x \, dx = \frac{2}{3}$. *(5 marks)*

7 The points A and B have coordinates $(1, 4, 2)$ and $(2, -1, 3)$ respectively.

The line l has equation $\mathbf{r} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$.

(a) Show that the distance between the points A and B is $3\sqrt{3}$. *(2 marks)*

(b) The line AB makes an acute angle θ with l. Show that $\cos\theta = \frac{7}{9}$. *(3 marks)*

(c) The point P on the line l is where $\lambda = p$.

(i) Show that

$$\overrightarrow{AP}.\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = 7 + 3p$$ *(4 marks)*

(ii) Hence find the coordinates of the foot of the perpendicular from the point A to the line l. *(3 marks)*

TURN OVER FOR THE NEXT QUESTION

Turn over ▶

8 (a) A cup of coffee is cooling down in a room. At time t minutes after the coffee is made, its temperature is x °C, where

$$x = 15 + 70e^{-\frac{t}{40}}$$

(i) Find the temperature of the coffee when it is made. *(1 mark)*

(ii) Find the temperature of the coffee 30 minutes after it is made. *(2 marks)*

(iii) Find how long it will take for the coffee to cool down to 60 °C. *(3 marks)*

(b) (i) Use integration to solve the differential equation

$$\frac{dx}{dt} = -\frac{1}{40}(x - 15) \ , \quad x > 15$$

given that $x = 85$ when $t = 0$, expressing t in terms of x. *(6 marks)*

(ii) Hence show that $x = 15 + 70e^{-\frac{t}{40}}$. *(2 marks)*

END OF QUESTIONS