LOGS AND EXPONENTIALS | TERMINOLOGY

base, exponent, "is equivalent to"

$$4^3 = 64 \Leftrightarrow \log_4 64 = 3$$

base

TIP: NOTICE HOW THE 'BASE' IS THE SAME IN BOTH FORMS.

- If no base is stated, we assume it is base 10
  - (i.e. $\log_{10} x = \log x$).
- The following applies to logarithms of any base:
  - $\log_a a = 1$ (e.g. $\log_2 2 = 1$)
  - $\log_a 1 = 0$

LOGS AND EXPONENTIALS | LOG LAWS

$$\log_a x + \log_a y = \log_a (xy) \quad \text{(addition law)}$$

$$\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right) \quad \text{(subtraction law)}$$

$$x \log_a y = \log_a (y^x) \quad \text{(power law)}$$

NOTICE THAT THE BASE MUST BE THE SAME FOR BOTH TERMS WHEN USING THE ADDITION AND SUBTRACTION LAW.

SOLVING LOG EQUATIONS | EXAMPLE PROBLEM PAIRS

1E. Solve $2^x = 5$.
Give your answer to 2 decimal places.

Method 1 – Equivalent Form:

$$2^x = 5 \Leftrightarrow x = \log_2 5$$
$$\therefore \ x = 2.32$$

Method 2 – Taking Logs

TAKE LOGS OF BOTH SIDES (ANY BASE)

$$2^x = 5$$
$$\log 2^x = \log 5$$

USE THE POWER LAW

$$x \log 2 = \log 5$$
$$x = \frac{\log 5}{\log 2}$$
$$= 2.32$$

1P. Solve $4^x = 9$
Give your answer to 2 decimal places.

Method 1 – Equivalent Form:

Method 2 – Taking Logs

2E. Solve $3^{2x-5} = 12$.
Give your answer to 2 decimal places.

Method 1 – Equivalent Form:

$$3^{2x-5} = 12 \Leftrightarrow 2x - 5 = \log_3 12$$
$$x = \frac{1}{2}\left(\log_3 12 + 5\right)$$
$$= 3.63$$

Method 2 – Taking Logs

$$3^{2x-5} = 12$$
$$\log\left(3^{2x-5}\right) = \log 12$$
$$(2x-5)\log 3 = \log 12$$

BRACKETS ARE REQUIRED HERE

$$2x - 5 = \frac{\log 12}{\log 3}$$
$$x = \frac{1}{2}\left(\frac{\log 12}{\log 3} + 5\right)$$
$$= 3.63$$

2P. Solve $5^{2-x} = 7$
Give your answer to 2 decimal places.

Method 1 – Equivalent Form:

Method 2 – Taking Logs

3E. Find the exact solution of $3(5)^x - 4 = 6$.

$$3(5)^x - 4 = 6$$
$$3(5)^x = 10$$
$$5^x = \frac{10}{3}$$
$$x = \log_5\left(\frac{10}{3}\right)$$

REARRANGE BEFORE TAKING LOGS

3P. Find the exact solution of $8(2)^{4x-3} = 7$.

4E. Find the exact solution of $2^{x+3} = 5^{x-1}$.

$$\log(2^{x+3}) = \log(5^{x-1})$$
$$(x+3)\log 2 = (x-1)\log 5$$
$$x\log 2 + 3\log 2 = x\log 5 - \log 5$$
$$3\log 2 + \log 5 = x\log 5 - x\log 2$$
$$3\log 2 + \log 5 = x\left(\log 5 - \log 2\right)$$
$$x = \frac{3\log 2 + \log 5}{\log 5 - \log 2}$$

COLLECT THE x-TERMS TOGETHER

EXPAND THE BRACKETS

FACTORISE AND DIVIDE

4P. Find the exact solution of $6^{x-2} = 3^x$.

5E. Solve $5^{2x} + 12 = 8(5^x)$.
Give your answer to 2 decimal places.

$$5^{2x} + 12 = 8(5^x)$$
$$5^{2x} - 8(5^x) + 12 = 0$$

Let $y = 5^x$

$$y^2 - 8y + 12 = 0$$
$$(y-6)(y-2) = 0$$
$$y = 6,\ y = 2$$

$\therefore\ 5^x = 6$ $\qquad$ $5^x = 2$

$x = \log_5 6$ $\qquad$ $x = \log_5 2$

$= 1.11$ $\qquad$ $= 0.43$

5P. Solve $3^{2x} - 5(3^x) + 4 = 0$.
Give your answer to 2 decimal places.