



DESCRIBING SEQUENCES | DEFINITIONS

- A recurrence relation describes how to get from one term to the next. This is sometimes called the term-to-term rule.
 - e.g. $2, 6, 10, 14, \dots \rightarrow u_1 = 2, u_{n+1} = u_n + 4$
- A position-to-term rule describes how to get any term in the sequence. This is sometimes called the nth term.
 - e.g. $2, 6, 10, 14, \dots \rightarrow u_n = 4n - 2$

DESCRIBING SEQUENCES | INCREASING AND DECREASING SEQUENCES

- An increasing sequence is one where each term is larger than the previous one.
 - $u_{n+1} > u_n$
- A decreasing sequence is one where each term is smaller than the previous one.
 - $u_{n+1} < u_n$
- A periodic sequence is one where the terms start repeating.
 - $u_{n+k} = u_n$ for some value of k , where k is called the period of the sequence.
- By finding $u_{n+1} - u_n$, we can prove if a sequence is increasing or decreasing.

DESCRIBING SEQUENCES | EXAMPLE PROBLEM PAIR 1

Find the first five terms of each sequence and describe the behaviour of the sequence.

(a) $x_{n+1} = 5x_n - 1, x_1 = 2$

(b) $u_n = \frac{16}{2^n}$

(c) $a_{n+1} = \frac{1}{a_n}, a_1 = 3$

Find the first five terms of each sequence and describe the behaviour of the sequence.

(a) $p_{n+1} = p^2 - p, p_1 = 3$

(b) $r_n = -\frac{2}{n^2}$ (tip: think about direction along a number line)

(b) $t_n = (-1)^n$

CONVERGENT SEQUENCES AND LIMITS | DEFINITION

- A sequence is said to be convergent if it approaches a specific value, called a limit.
 - e.g. As $n \rightarrow \infty$, the sequence $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$ approaches the limit $L = 0$. This is a convergent sequence.



CONVERGENT SEQUENCES AND LIMITS | EXAMPLE PROBLEM PAIR

A convergent sequence is defined by $u_{n+1} = \frac{2}{3}u_n + 5$, $u_1 = 6$.

Find the limit, L , as $n \rightarrow \infty$.

A convergent sequence is defined by $u_{n+1} = \frac{1}{4}u_n + 3$, $u_1 = 5$.

Find the limit, L , as $n \rightarrow \infty$.

DESCRIBING SEQUENCES | PROOF QUESTIONS

The n th term of a sequence is defined by $u_n = 2n - n^2$.

Prove that the sequence is decreasing.

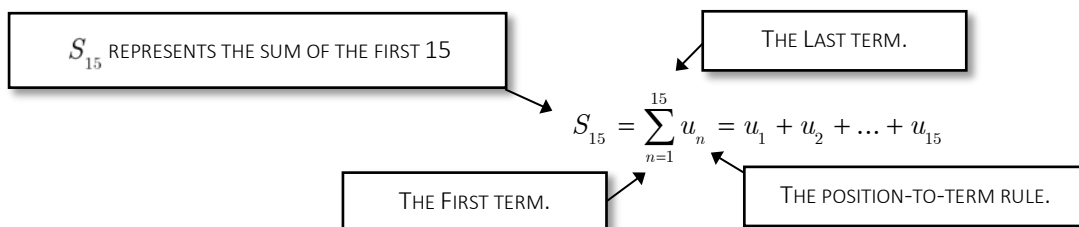
A sequence has n th term $u_n = 7n - 16$.

Show that the sequence is increasing.

SIGMA NOTATION | KEY FACTS

» Sigma notation ($\sum$) can be used as an alternative way to describe a series.

o The example below means 'sum the first 15 terms of the sequence described by the position to term rule u_n '.



SIGMA NOTATION | EXAMPLE PROBLEM PAIR

Find $\sum_{k=1}^7 (2k + 1)$

Find $\sum_{r=5}^{10} r^2$
