BINOMIAL EXPANSIONS

lesson link: parkermaths.com/y1binomial

Objective

- Understand and use the binomial expansion of $(ax+b)^n$ for positive integer n.

2.1 Key Facts: Informal Binomial Expansion

- A **binomial** is a polynomial that is the **sum of two terms** (e.g. $ax + by$).
- We can either use the **binomial formula** or **Pascal's triangle** to expand expressions of the form $(a+b)^n$.

Pascal's Triangle

Pascal's triangle is made up of the binomial coefficients.

Row 0 → 1

1 1

1 2 1

1 3 3 1 ← Row 3

1 4 6 4 1

1 5 10 10 5 1

Examples: Simple Binomial Expansions

2.2e. Expand $(x+y)^4$.

$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

2.2p. Expand $(x+y)^5$.

2.3e. Expand $(x+2)^5$.

$$(x+2)^5 = x^5 + 5x^4(2) + 10x^3(2)^2 + 10x^2(2)^3 + 5x(2)^4 + 2^5$$
$$= x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$$

2.3p. Expand $(y+3)^4$.

2.4e. Expand $(3-2y)^4$.

$$(3-2y)^4 = 3^4 + 4(3)^3(-2y) + 6(3)^2(-2y)^2 + 4(3)(-2y)^3 + (-2y)^4$$
$$= 81 + 4(27)(-2y) + 6(9)(4y^2) + 4(3)(-8y^3) + 16y^4$$
$$= 81 - 216y + 216y^2 - 96y^3 + 16y^4$$

2.4p. Expand $(3x-2)^3$.

3.1 Key Facts: Factorial Notation

DEFINITION	EXAMPLE	PRONUNCIATION
$n! = n\times(n-1)\times(n-2)\times\ldots\times3\times2\times1$	$5! = 5\times4\times3\times2\times1 = 120$	'n factorial'

Quickfire Questions

FACTORIAL REPRESENTATION	EXPANDED REPRESENTATION	SIMPLIFIED/EVAULATED
$\frac{6!}{3!}$		
	$3\times2\times1\times4\times3\times2\times1$	
$\frac{2!6!}{9!}$		
$(n-2)!$		N/A
	$(r-4)\times(r-5)\times(r-6)\times\ldots\times3\times2\times1$	N/A
$(p+1)!$		N/A
$\frac{n!}{(n-2)!}$		

3.2 Key Facts: Binomial Coefficients

Binomial coefficient can be calculated as follows:

Row → n, Entry → r

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

e.g. $\binom{5}{3} = {}^{5}C_{3} = \frac{5!}{3!(5-3)}$

$= \frac{5!}{3!2!}$

$= 10$

In terms of binomial coefficients

$$\binom{0}{0}$$
$$\binom{1}{0} \quad \binom{1}{1}$$
$$\binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$$
$$\binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$$
$$\binom{4}{0} \quad \binom{4}{1} \quad \binom{4}{2} \quad \binom{4}{3} \quad \binom{4}{4}$$

Top tips:

- Rows and entries start at 0.
- Your calculator can evaluate binomial coefficients directly.
- Notice the symmetry in Pascal's triangle
  - o E.g. $\binom{5}{1} = \binom{5}{4} = 5$

3.4 Quickfire Questions

(a) Use your calculator to evaluate the following:

(i) $\binom{8}{3} =$ (ii) ${}^{15}C_{5} =$ (iii) ${}^{15}C_{2} =$

(b) Use your answers to part (a) to write down the value of:

(i) ${}^{15}C_{13} =$ (ii) $\binom{15}{10} =$

f(x) BINOMIAL FORMULA (IN FORMULA BOOKLET)

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \quad (n \in \mathbb{N}) \qquad \text{where } \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Examples: Binomial Expansion (Formula)

3.5e. Find and simplify the first 5 terms in the expansion of $(2-x)^8$ in ascending powers of x.

$$(2-x)^8 = 2^8 + \binom{8}{1}(2)^7(-x)^1 + \binom{8}{2}(2)^6(-x)^2 + \binom{8}{3}(2)^5(-x)^3 + \binom{8}{4}(2)^4(-x)^4 + \ldots$$

$$= 256 + 8(128)(-x) + 28(64)(x^2) + 56(32)(-x^3) + 70(16)(x^4) + \ldots$$

$$= 256 - 1024x + 1792x^2 - 1792x^3 + 1120x^4$$

3.5p. Find and simplify the first 4 terms in the expansion of $(1-2x)^9$ in ascending powers of x.

Example: Finding a Single Term

3.6e. Find the coefficient of x^6 in the expansion of $(2x+5)^8$

x^6 term: $\binom{8}{2}(2x)^6(5)^2 = 28(64x^6)(25)$

$= 44800x^6$

$\therefore$ coefficient of x^6 is 44800

3.6p. Find the coefficient of x^2 in the expansion of $(3x-4)^7$

3.7 Quickfire Questions

Expression	Power of x	Term in expansion (show working out)
$(x-3)^{10}$	x^7	
$(x+2)^{20}$	x^{13}	
$(5-3x)^9$	x^8	
$(2x+a)^{14}$	x^6	