

EQUATION OF A CIRCLE



Objective

- Understand and use the equation of a circle.
- Find the equation of a circle given two points that form a diameter.
- To sketch a circle given the equation of the circle.
- To determine the location of a point relative to a circle (inside, on or outside).



KEY FACT

The circle with centre (a, b) and radius r has equation $(x - a)^2 + (y - b)^2 = r^2$

Examples

2.2e. Write down the equation of a circle with

- (a) centre $(2, 5)$, radius 8,
- (b) centre $(-3, 1)$, radius 5.

$$(a) \quad (x - 2)^2 + (y - 5)^2 = 64$$

$$(b) \quad (x + 3)^2 + (y - 1)^2 = 25$$

2.3e. Find the centre and radius of the circle with equation

$$x^2 + y^2 - 12x + 6y - 4 = 0.$$

$$x^2 - 12x + y^2 + 6y - 4 = 0$$

$$(x - 6)^2 - 36 + (y + 3)^2 - 9 - 4 = 0$$

$$(x - 6)^2 + (y + 3)^2 = 49$$

Complete the square for x and y separately.

Centre $(6, -3)$

Radius = 7

2.4e. **Verify** that the point $(3, 8)$ does **not** lie on the circle with equation

$$x^2 + y^2 - 4x + 6y - 20 = 0.$$

When $x = 3$ and $y = 8$,

$$\text{LHS} = 3^2 + 8^2 - 4(3) + 6(8) - 20$$

$$= 9 + 64 - 12 + 48 - 20$$

$$= 89$$

$\therefore \text{LHS} \neq \text{RHS}$

\therefore The point does not lie on the circle.

2.2p. Write down the equation of a circle with

- (a) centre $(4, -5)$, radius 3,
- (b) centre $(0, 8)$, radius 7.

2.3p. Find the centre and radius of the circle with equation

$$x^2 + y^2 + 10x + 4y - 71 = 0$$

2.4p. **Verify** that the point $(-1, -5)$ lies on the circle with equation

$$x^2 + y^2 + 8x + 2y - 8 = 0.$$
