Y1 DIFFERENTIATION 1

lesson link: parkermaths.com/y1diff

Objective

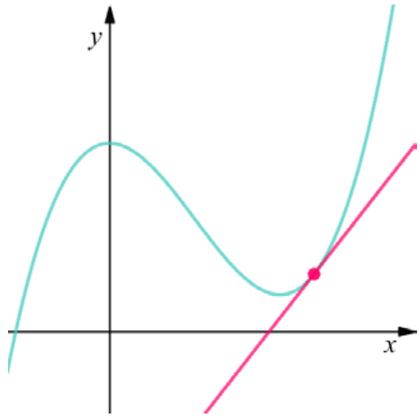
- Understand and use the gradient function.
- To differentiate functions of the form $f(x) = a$, $f(x) = kx$ and $f(x) = x^n$, where a and k are constants.

2.1 The Gradient Function

The **gradient** at a given point is defined as the gradient of the **tangent** to the curve at that point.

The **gradient function**, or **derivative**, of the curve $y = f(x)$, is written as

$$\frac{dy}{dx} \quad \text{or} \quad f'(x)$$



2.2 Rules for Differentiation

Function	Gradient Function (Derivative)	Example
$y = a$	$\frac{dy}{dx} = 0$	$y = 5$, $\frac{dy}{dx} = 0$
$y = kx$	$\frac{dy}{dx} = k$	$y = 3x$, $\frac{dy}{dx} = 3$
$y = x^n$	$\frac{dy}{dx} = nx^{n-1}$	$y = x^3$, $\frac{dy}{dx} = 3x^2$

The gradient of a horizontal line is 0.

$y = kx$ is a straight line with gradient k.

Examples: Rules for Differentiation

2.2e.

(a) Differentiate $y = x^5$

multiply by the power

$\frac{dy}{dx} = 5x^4$

subtract 1 from the power

(b) $f(x) = x^{-2}$. Find $f'(x)$.

$f'(x) = -2x^{-3}$

$= -\frac{2}{x^3}$

This last step it not strictly necessary, but it is often useful to rewrite your answer in this form

(c) $\frac{d}{dx} x^{\frac{1}{3}}$

$\frac{d}{dx} x^{\frac{1}{3}} = \frac{1}{3} x^{-\frac{2}{3}}$

$= \frac{1}{3x^{\frac{2}{3}}}$

2.2p.

(a) Differentiate $y = x^7$

(b) $f(x) = x^{\frac{7}{5}}$. Find $f'(x)$.

(c) $\frac{d}{dx} x^{-4}$

2.3e. (a) Find the gradient of the curve $y = x^2$ at the where point $x = -2$.

This means 'evaluate the derivative at $x = -2$'

$$\frac{dy}{dx} = 2x$$

$$\left.\frac{dy}{dx}\right|_{x=-2} = 2(-2)$$

$$= -4$$

(b) Find the gradient of the curve $f(x) = x^{\frac{1}{2}}$ at the point where $x = 9$.

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2x^{\frac{1}{2}}}$$

$$f'(9) = \frac{1}{2(9)^{\frac{1}{2}}} = \frac{1}{6}$$

2.3e. (a) Find the gradient of the curve $y = x^4$ at the where point $x = 1$.

(b) Find the gradient of the curve $f(x) = x^{-1}$ at the point where $x = 3$.

2.4 Rules for Differentiation

$$\frac{d}{dx}kf(x) = k\frac{d}{dx}f(x) \quad , \quad \frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

Examples: Rules for Differentiation

2.4e. Find the derivative of

(a) $f(x) = \frac{1}{2}x^3$

$$f'(x) = \frac{3}{2}x^3$$

(b) $y = x^{\frac{1}{2}} + x^{-3} - 2$

$$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - 3x^{-4}$$

(c) $f(x) = 2x^3 - 4x^{\frac{3}{2}} - 9x + 2x^{-1}$

$$f'(x) = 6x^2 - 6x^{\frac{1}{2}} - 9 - 2x^{-2}$$

2.5e. (a) $f(x) = 6\sqrt{x} - \frac{5}{x^4}$. Find $f'(x)$.

$$f(x) = 6x^{\frac{1}{2}} - 5x^{-4}$$

We first 'prepare' the equation for differentiation.

$$f'(x) = 3x^{-\frac{1}{2}} + 20x^{-5}$$

(b) $y = 2x\sqrt{x} - \frac{8}{\sqrt[3]{x}} + \frac{2}{3x}$. Find $\frac{dy}{dx}$.

$$y = 2x^{\frac{3}{2}} - 8x^{-\frac{1}{3}} + \frac{2}{3}x^{-1}$$

$$\frac{dy}{dx} = 3x^{\frac{1}{2}} + \frac{8}{3}x^{-\frac{4}{3}} - \frac{2}{3}x^{-2}$$

2.4e. Find the derivative of

(a) $f(x) = 2x^5$

(b) $y = x^{-2} + x^{\frac{1}{4}} + 6x$

(c) $f(x) = \frac{1}{4}x^2 - 3x^{\frac{5}{2}} + x^{\frac{1}{3}} + 5$

2.5e. (a) $y = \sqrt[3]{x} - \frac{2}{x}$. Find $\frac{dy}{dx}$.

(b) $f(x) = \frac{4}{\sqrt{x}} - x^2\sqrt{x} + \frac{7x}{3\sqrt{x}}$. Find $f'(x)$.