

SOLVING QUADRATIC EQUATIONS



Objective

- To solve quadratic equations by factorising, using the formula and directly using your calculator.

Examples

4.1e. Solve the following equations by factorising.

(a) $(4x-9)(5-3x) = 0$

$4x - 9 = 0$ or $5 - 3x = 0$

$4x = 9$

$x = \frac{9}{4}$

$5 = 3x$

$x = \frac{5}{3}$

If you are comfortable doing this in your head, there is no need to show working.

(b) $5w = 2w^2$

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$0 = 2w^2 - 5w$

$0 = w(2w - 5)$

Do not 'cancel' or divide by w if $w = 0$ might be a solution (rearrange and factorise instead).

$w = 0$ or $2w - 5 = 0$

$2w = 5$

$w = \frac{5}{2}$

(c) $15t^2 + 25t = 40$

$15t^2 + 25t = 40$

$15t^2 + 25t - 40 = 0$

$3t^2 + 5t - 8 = 0$

$(3t + 8)(t - 1) = 0$

Check for common factors before factorising.

Use your preferred method to factorise.

$3t + 8 = 0$ or $t - 1 = 0$

$3t = -8$

$t = 1$

$t = -\frac{8}{3}$

Following this step you can just write down the answers if you feel comfortable doing so.

4.1p. Solve the following equations by factorising.

(a) $(1-6x)(8x-5) = 0$

(b) $7w - 4w^2 = 0$

(c) $4t^2 = 38t - 90$

**KEY FACT**

The solution of $ax^2 + bx + c = 0$, where $a \neq 0$, is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Examples

4.2e. Find the exact solutions to the equation

$$2x + 7 = (3x + 2)^2$$

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$$2x + 7 = 9x^2 + 12x + 4$$

$$0 = 9x^2 + 10x - 3$$

$$\therefore x = \frac{-10 \pm \sqrt{10^2 - 4 \times 9 \times (-3)}}{2 \times 9}$$

$$= \frac{-10 \pm \sqrt{208}}{18}$$

$$= \frac{-5 \pm 2\sqrt{13}}{9}$$

← Leave your answer in exact form.

4.3e. Use your calculator to find the exact solutions to the equation

$$7 + 2y^{-1} = 3y$$

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$$7y + 2 = 3y^2$$

$$0 = 3y^2 - 7y - 2$$

$$y = \frac{7 \pm \sqrt{73}}{6} \quad (\text{BC})$$

← BC stands for 'BC' calculator. It indicates to the reader why there is no working out.

4.4e. Solve the equation $x^2 = 10kx - 21k^2$, giving your answer in terms of k .

$$x^2 = 10kx - 21k^2$$

$$x^2 - 10kx + 21k^2 = 0$$

$$\therefore x = \frac{10k \pm \sqrt{(10k)^2 - 4 \times 1 \times (-21k^2)}}{2}$$

$$= \frac{10k \pm \sqrt{16k^2}}{2}$$

$$= \frac{10k \pm 4k}{2}$$

$$\therefore x = 7k \quad \text{or} \quad x = 3k$$

4.2e. Find the exact solutions to the equation

$$\frac{3x + 2}{x + 2} = \frac{4x + 1}{6 - x}$$

4.3e. Use your calculator to find the exact solutions to the equation

$$3y + 13 + \frac{6}{y} = 0$$

4.4e. Solve the equation $x^2 - 2px = 8p^2$, giving your answer in terms of p .
