



Objective

- Use and manipulate surds, including rationalising the denominator.



KEY FACTS

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

$$\sqrt{a^2} = (\sqrt{a})^2 = a$$

Examples

2.2e. Simplify the following:

(a) $\sqrt{48}$

$$\sqrt{48} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$$

(b) $3\sqrt{8} + 4\sqrt{2}$

$$\begin{aligned} 3\sqrt{8} + 4\sqrt{2} &= 3 \times \sqrt{4} \times \sqrt{2} + 4\sqrt{2} \\ &= 6\sqrt{2} + 4\sqrt{2} \\ &= 10\sqrt{2} \end{aligned}$$

2.3e. Simplify the following:

(a) $5\sqrt{5} \times 4\sqrt{80}$

$$\begin{aligned} 5\sqrt{5} + 4\sqrt{80} &= 20\sqrt{5} \times \sqrt{80} \\ &= 20\sqrt{5} \times \sqrt{16} \times \sqrt{5} \\ &= 20 \times 5 \times 4 \\ &= 400 \end{aligned}$$

(b) $\sqrt{\frac{18}{25}}$

$$\begin{aligned} \sqrt{\frac{18}{25}} &= \frac{\sqrt{18}}{\sqrt{25}} \\ &= \frac{3\sqrt{2}}{5} \\ &= \frac{3\sqrt{2}}{5} \end{aligned}$$

2.4e. Expand and simplify:

(a) $(4 + 3\sqrt{6})(5\sqrt{6} - 1)$

$$\begin{aligned} (4 + 3\sqrt{6})(5\sqrt{6} - 1) &= 20\sqrt{6} - 4 + 15 \times 6 - 3\sqrt{6} \\ &= 17\sqrt{6} + 84 \end{aligned}$$

(b) $(3 - 4\sqrt{7})^2$

$$\begin{aligned} (3 - 4\sqrt{7})^2 &= 9 - 2 \times 3 \times 4\sqrt{7} + 16 \times 7 \\ &= 121 - 24\sqrt{7} \end{aligned}$$

2.2p. Simplify the following:

(a) $\sqrt{300}$

(b) $2\sqrt{45} - \sqrt{20}$

2.3p. Simplify the following:

(a) $4\sqrt{28} \times 5\sqrt{7}$

(b) $\sqrt{\frac{24}{9}}$

2.4p. Simplify the following:

(a) $(8 + \sqrt{2})(3\sqrt{2} + 5)$

(b) $(5 - 2\sqrt{3})^2$

2.5e. Rationalise the denominator of:

(a) $\frac{2}{\sqrt{5}}$

$$\frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

(b) $\frac{9}{\sqrt{3}}$

$$\begin{aligned} \frac{9}{\sqrt{3}} &= \frac{9}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{9\sqrt{3}}{3} \\ &= 3\sqrt{3} \end{aligned}$$

(c) $\frac{4}{3\sqrt{6}}$

$$\begin{aligned} \frac{4}{3\sqrt{6}} &= \frac{4}{3\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} \\ &= \frac{4\sqrt{6}}{18} \\ &= \frac{2\sqrt{6}}{9} \end{aligned}$$

2.5p. Rationalise the denominator of:

(a) $\frac{5}{\sqrt{7}}$

(b) $\frac{8}{5\sqrt{2}}$

(c) $\frac{21}{\sqrt{7}}$

2.6e. (a) Expand and simplify $(2+\sqrt{7})(2-\sqrt{7})$.

$$\begin{aligned} (2+\sqrt{7})(2-\sqrt{7}) &= 4 - 2\sqrt{7} + 2\sqrt{7} - 7 \\ &= -3 \end{aligned}$$

(b) Rationalise the denominator of $\frac{3}{8-2\sqrt{3}}$.

$$\begin{aligned} \frac{3}{8-2\sqrt{3}} &= \frac{3}{8-2\sqrt{3}} \times \frac{8+2\sqrt{3}}{8+2\sqrt{3}} \\ &= \frac{24+6\sqrt{3}}{64-12} \\ &= \frac{24+6\sqrt{3}}{52} \\ &= \frac{12+3\sqrt{3}}{26} \end{aligned}$$

(c) Express $\frac{9\sqrt{2}-8}{3\sqrt{2}-5}$ in the form $a+b\sqrt{2}$.

$$\begin{aligned} \frac{9\sqrt{2}-8}{3\sqrt{2}-5} &= \frac{9\sqrt{2}-8}{3\sqrt{2}-5} \times \frac{3\sqrt{2}+5}{3\sqrt{2}+5} \\ &= \frac{27 \times 2 + 45\sqrt{2} - 24\sqrt{2} - 40}{9 \times 2 - 25} \\ &= \frac{14 + 21\sqrt{2}}{-7} \\ &= -2 - 3\sqrt{2} \end{aligned}$$

2.6p. (a) Expand and simplify $(5+\sqrt{3})(5-\sqrt{3})$.

(b) Rationalise the denominator of $\frac{4}{1+\sqrt{3}}$.

(c) Express $\frac{7\sqrt{3}-2}{3\sqrt{3}-4}$ in the form $p+q\sqrt{3}$.
