

 Key Facts

- When **one vertical transformation** is combined with **one horizontal transformation**...
- When **two horizontal transformations** or **two vertical transformations** are combined...

2.2e. The graph $y = \frac{1}{x}$ is translated by $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$, followed by a stretch of scale factor of 2 in the y -direction.

Find the equation of the transformed graph.

Apply transformations
one at a time

Translate by $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$: ← 1

$$y = \frac{1}{x} \rightarrow y = \frac{1}{x-3}$$

Stretch, s.f. 2 in y -direction: ← 2

$$y = \frac{1}{x-3} \rightarrow \frac{y}{2} = \frac{1}{x-3}$$

$$y = \frac{2}{x-3}$$

2.2p. The graph $y = \sin x$ is translated by 2 units in the negative y -direction and stretched by a scale factor of 3 in the x -direction.

Find the equation of the transformed graph.

2.3e. Describe a sequence of transformations which maps the graph of $y = e^x$ onto the graph $y = e^{2x} + 5$.

Translation $\begin{pmatrix} 0 \\ 5 \end{pmatrix}$: *Show the working out for each stage.*

$$y = e^x \rightarrow y - 5 = e^x$$

$$y = e^x + 5$$

Stretch, s.f. $\frac{1}{2}$ parallel to the x-axis:

$$y = e^x - 5 \rightarrow y = e^{2x} - 5$$

2.3p. Describe a sequence of transformations which maps the graph of $y = \ln x$ onto the graph $y = 3\ln(x + 4)$.

2.5e. A graph has equation $y = x^3$. Find the equation of the graph after the following transformations:

- (a) A translation of $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$, followed by a stretch parallel to the y-axis with scale factor 4.
- (b) A stretch parallel to the y-axis with scale factor 4, followed by a translation of $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$.

(a) Translation $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$:

$$y = x^3 \rightarrow y = (x - 3)^3$$

Horizontal stretch, s.f. 4:

$$y = (x - 3)^3 \rightarrow y = \left(\frac{1}{4}x - 3\right)^3$$

(b) Horizontal stretch, s.f. 4:

$$y = x^3 \rightarrow y = \left(\frac{1}{4}x\right)^3$$

Translation $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$:

$$y = \left(\frac{1}{4}x\right)^3 \rightarrow y = \left(\frac{1}{4}(x - 3)\right)^3$$

$$y = \left(\frac{1}{4}x - \frac{3}{4}\right)^3$$

2.5p. A graph has equation $y = 2^x$. Find the equation of the graph after the following transformations:

- (a) A vertical stretch with scale factor 5, following by a translation of $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$.
- (b) A translation of $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$, followed by a vertical stretch with scale factor 5.
