VIDEO PLAYLIST: <link>

OPTIMISATION ▸ INTRODUCTION

- To solve optimisation problems, apply your knowledge about stationary points to determine minimum and maximum points in the context of real-world problems.
- Try to **model** the problem with a **diagram**. If you have two variables, try to eliminate one using substitution.

OPTIMISATION ▸ EXAMPLE-PROBLEM PAIRS

1E. A wire of length 12 cm is bent to form a rectangle.

(a) Show that the area, A is given by $A = 6x - x^2$, where x is the width of the rectangle.

(b) Find the maximum possible area and justify that it is a maximum.

(a) Perimeter: $12 = 2x + 2y$

$6 = x + y$

$y = 6 - x$

Area: $A = xy$

Substitute (1) into (2):

$A = x(6 - x)$

$= 6x - x^2$

(b) $$\frac{dA}{dx} = 6 - 2x$$

Stationary point when $\frac{dA}{dx} = 0$.

$\therefore\ 0 = 6 - 2x$

$x = 3$

Find the stationary point.

$$\frac{d^2A}{dx^2} = -2 < 0$$

Justify it is a maximum.

$\therefore$ Maximum area $= 6(3) - (3)^2$

$= 9\ \text{cm}^2$

1P. A farmer wants to build a rectangular sheep pen with length L.

She has 20 m of fencing in total and wants the area inside the pen to be as large as possible.

(a) Show that the area, A is given by $A = 10L - L^2$, where L is the length of the rectangle.

(b) Find the length and width of the pen that give the maximum area.

2E. An open-topped cylindrical can has base radius r and height h.

The external surface area $A = 243\pi\ \text{cm}^2$. .

(a) Show that the volume of the can is given by

$$V = \frac{\pi}{2}\left(243r - r^3\right).$$

(b) Hence find the maximum capacity of the can.

(a) Volume of can: $V = \pi r^2 h$

Surface area: $243\pi = \pi r^2 + 2\pi r h$

$243 = r^2 + 2rh$

$h = \dfrac{243 - r^2}{2r}$

$\therefore \quad V = \pi r^2\left(\dfrac{243 - r^2}{2r}\right)$

$= \dfrac{\pi}{2}\left(243r - r^3\right)$

(b)

$V = \dfrac{\pi}{2}\left(243r - r^3\right)$

$V = \dfrac{243\pi}{2}r - \dfrac{\pi}{2}r^3$

$\dfrac{dV}{dr} = \dfrac{243\pi}{2} - \dfrac{3\pi}{2}r^2$

Min/Max when $\dfrac{dV}{dr} = 0$

$\therefore \quad 0 = \dfrac{243\pi}{2} - \dfrac{3\pi}{2}r^2$

$\dfrac{3\pi}{2}r^2 = \dfrac{243\pi}{2}$

$r^2 = 81$

$r = \pm 9$

Since $r > 0$, $r = 9$.

$\dfrac{d^2V}{dr^2} = -3\pi r$

$\left.\dfrac{d^2V}{dr^2}\right|_{r=9} = -27\pi < 0$

$\therefore \quad r = 9$ is a maximum point.

$\therefore \quad V_{max} = \dfrac{\pi}{2}\left(243 \times 9 - (9)^3\right)$

$= 729\pi\ cm^3$

2P. An open-topped cylindrical pie tin is t cm high with a radius of r cm.

The volume of the pie tin is $1000\ \text{cm}^3$. .

(a) Show that the surface area of the tin is given by

$$A = \pi r^2 + \frac{2000}{r}.$$

(b) Find the minimum surface area.