



LOGS AND EXPONENTIALS | TERMINOLOGY

base $\swarrow$ $4^3 = 64 \Leftrightarrow \log_4 64 = 3$ $\nwarrow$ exponent "is equivalent to"
base

TIP: NOTICE HOW THE 'BASE' IS THE SAME IN BOTH FORMS.

- If no base is stated, we assume it is base 10
 - o (i.e. $\log_{10} x = \log x$).
- The following applies to logarithms of any base:
 - o $\log_a a = 1$ (e.g. $\log_2 2 = 1$)
 - o $\log_a 1 = 0$

LOGS AND EXPONENTIALS | LOG LAWS

$$\log_a x + \log_a y = \log_a (xy) \quad (\text{addition law})$$

$$\log_a x - \log_a y = \log_a \left(\frac{x}{y} \right) \quad (\text{subtraction law})$$

$$x \log_a y = \log_a (y^x) \quad (\text{power law})$$

NOTICE THAT THE BASE MUST BE THE SAME FOR BOTH TERMS WHEN USING THE ADDITION AND SUBTRACTION LAW.

SOLVING LOG EQUATIONS | EXAMPLE PROBLEM PAIRS

1E. Solve $2^x = 5$.

Give your answer to 2 decimal places.

Method 1 – Equivalent Form:

$$2^x = 5 \Leftrightarrow x = \log_2 5$$

$$\therefore x = 2.32$$

Method 2 – Taking Logs

$$2^x = 5$$

$$\log 2^x = \log 5$$

$$x \log 2 = \log 5$$

$$x = \frac{\log 5}{\log 2}$$

$$= 2.32$$

TAKE LOGS OF BOTH SIDES (ANY BASE)

USE THE POWER LAW

1P. Solve $4^x = 9$

Give your answer to 2 decimal places.

Method 1 – Equivalent Form:

Method 2 – Taking Logs

2E. Solve $3^{2x-5} = 12$.

Give your answer to 2 decimal places.

Method 1 – Equivalent Form:

$$3^{2x-5} = 12 \Leftrightarrow 2x - 5 = \log_3 12$$

$$x = \frac{1}{2} (\log_3 12 + 5)$$

$$= 3.63$$

Method 2 – Taking Logs

$$3^{2x-5} = 12$$

$$\log(3^{2x-5}) = \log 12$$

$$(2x - 5) \log 3 = \log 12$$

$$2x - 5 = \frac{\log 12}{\log 3}$$

$$x = \frac{1}{2} \left(\frac{\log 12}{\log 3} + 5 \right)$$

$$= 3.63$$

BRACKETS ARE REQUIRED HERE

2P. Solve $5^{2-x} = 7$

Give your answer to 2 decimal places.

Method 1 – Equivalent Form:

Method 2 – Taking Logs

3E. Find the exact solution of $3(5)^x - 4 = 6$.

$$3(5)^x - 4 = 6$$

$$3(5)^x = 10$$

$$5^x = \frac{10}{3}$$

$$x = \log_5 \left(\frac{10}{3} \right)$$

REARRANGE
BEFORE
TAKING LOGS

4E. Find the exact solution of $2^{x+3} = 5^{x-1}$.

$$\log(2^{x+3}) = \log(5^{x-1})$$

$$(x+3)\log 2 = (x-1)\log 5$$

$$x\log 2 + 3\log 2 = x\log 5 - \log 5$$

$$3\log 2 + \log 5 = x\log 5 - x\log 2$$

$$3\log 2 + \log 5 = x(\log 5 - \log 2)$$

$$x = \frac{3\log 2 + \log 5}{\log 5 - \log 2}$$

COLLECT THE
 x -TERMS
TOGETHER

EXPAND THE
BRACKETS

FACTORISE
AND DIVIDE

3P. Find the exact solution of $8(2)^{4x-3} = 7$.

4P. Find the exact solution of $6^{x-2} = 3^x$.

SOLVING LOG EQUATIONS WITH HIDDEN QUADRATICS | EXAMPLE PROBLEM PAIR

5E. Solve $5^{2x} + 12 = 8(5^x)$.

Give your answer to 2 decimal places.

$$5^{2x} + 12 = 8(5^x)$$

$$5^{2x} - 8(5^x) + 12 = 0$$

$$\text{Let } y = 5^x$$

$$y^2 - 8y + 12 = 0$$

$$(y-6)(y-2) = 0$$

$$y = 6, y = 2$$

$$\therefore 5^x = 6$$

$$x = \log_5 6$$

$$= 1.11$$

$$5^x = 2$$

$$x = \log_5 2$$

$$= 0.43$$

5P. Solve $3^{2x} - 5(3^x) + 4 = 0$.

Give your answer to 2 decimal places.
