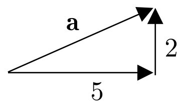
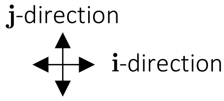
Year 1 | Vectors 1

Vector Notation | Key Facts

- A *scalar* has only *magnitude* (size). A *vector* has *magnitude and direction*.
- Vectors can be expressed as column vectors, or using $\mathbf{i},\mathbf{j}$ notation:

Column Vector Notation → $\mathbf{a} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} = 5\mathbf{i} + 2\mathbf{j}$ ← $\mathbf{i},\mathbf{j}$ notation.





Magnitude and Direction | Key Facts

- The *magnitude* of a vector is its length. If $\mathbf{a} = \begin{pmatrix} p \\ q \end{pmatrix}$ then $|\mathbf{a}| = \sqrt{p^2 + q^2}$
- The *direction* of a vector is the angle it makes with a specified direction.

Magnitude and direction | Example-Problem Pairs

Find the exact magnitude of the vector $\mathbf{a} = \begin{pmatrix} -4 \\ 7 \end{pmatrix}$ and the angle it makes with the direction of $\mathbf{i}$.

Find the exact magnitude of the vector $\mathbf{b} = 5\mathbf{i} + 3\mathbf{j}$ and the angle it makes with the direction of $\mathbf{i}$.

Vector $\mathbf{v}$ has a magnitude 7 and makes an angle of $62°$ below the $\mathbf{i}$ - direction. Write $\mathbf{v}$ as a column vector.

Given a vector $\mathbf{w} = a\mathbf{i} + b\mathbf{j}$, with direction $30°$ and magnitude 5, calculate $\mathbf{a}$ and $\mathbf{b}$.

Vector Operations | Key Facts

- Given two vectors, $\mathbf{p} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$, we can perform the following vector operations:
  - e.g. $\mathbf{p} + \mathbf{q} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} + \begin{pmatrix} -2 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 + (-2) \\ 5 + 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 9 \end{pmatrix}$ (vector *addition/subtraction*)
  - e.g. $\mathbf{p} = 8\begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 8 \times 3 \\ 8 \times 5 \end{pmatrix} = \begin{pmatrix} 24 \\ 40 \end{pmatrix}$ (scalar *multiplication*.)
- Vector multiplication is covered on the A-Level Further Mathematics course.

Equal and Parallel Vectors | Key Facts

- Two vectors are *equal* if they have the *same magnitude and direction* (they do not need to occupy the same position in space).
- If $\mathbf{a}$ and $\mathbf{b}$ are *parallel*, they can be written as $\mathbf{b} = t\mathbf{a}$, for some scalar t.
- A *unit vector* is a vector with a magnitude of one unit. To find a unit vector in the direction of a particular vector, divide the vector by its magnitude.

Parallel Vectors | Example-Problem Pairs

Given that $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -3 \\ p \end{pmatrix}$, and $\mathbf{c} = \begin{pmatrix} 5 \\ 10 \end{pmatrix}$.

(a) Show that $\mathbf{a}$ is parallel to $\mathbf{c}$.

(b) Find the value of p such that $\mathbf{b}$ is parallel to $\mathbf{a}$.

Find two unit vectors parallel to $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

Find the unit vector in the direction of $\mathbf{q} = 5\mathbf{i} - 12\mathbf{j}$.

Displacement and Position Vectors | Key Facts

- A vector that represents the *displacement between two points* is called a *displacement vector*.
- A vector that represents the *displacement from a fixed origin* is called a *position vector*.
- If points A and B have *position vectors* $\mathbf{a}$ and $\mathbf{b}$, then $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$.

Displacement and Position Vectors | Example-Problem Pairs

(a) Find the distance between points A and B with position vectors $\mathbf{a} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$.

(b) Point C has position vector $c = \begin{pmatrix} 2 \\ p \end{pmatrix}$.

Find the exact value of p such that $AC = 3AB$.

Points P and Q have position vectors $\overrightarrow{OP} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$ and $\overrightarrow{OQ} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$.

Find the distance PQ.