THE CHAIN RULE | KEY FACTS

- The chain rule is used to differentiate composite functions.
  - i.e. functions within functions
- To apply the chain rule, we need to know how to differentiate each part of the composite function.
- For simple cases, you should aim to be able to do the chain rule in your head.

THE CHAIN RULE

If $y = \mathrm{f}(u)$, where $u = \mathrm{g}(x)$, then

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x}$$

THE CHAIN RULE | EXAMPLE PROBLEM PAIRS

1E. $y = (2x-5)^6$. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

1P(a). $y = (3x+1)^5$. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

1P(b). $y = \left(\frac{1}{3}x + 1\right)^4$. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

2E. Find the derivative of $y = 4(2x^2 - 3x + 7)^3$

2P (a). Find the derivative of $y = 3(5x^2 + 2x + 3)^5$

2P (b). Find the derivative of $y = \frac{2}{7}\left(x^4 + \frac{1}{3}x^2\right)^4$

3E. Find $\frac{\mathrm{d}}{\mathrm{d}x}\sqrt{3x+1}$

3P (a). Find $\frac{\mathrm{d}}{\mathrm{d}x}\sqrt[3]{5x-6}$

3P (b). Find $\frac{\mathrm{d}}{\mathrm{d}x}\frac{3}{\sqrt{4x+9}}$

4E. $\mathrm{f}(x) = 3e^{5x} + 7$. Find $\mathrm{f}'(x)$.

4P (a). $\mathrm{f}(x) = 2e^{3x} - 2$. Find $\mathrm{f}'(x)$.

4P (b). $\mathrm{f}(x) = 2e^{8-3x^2}$. Find $\mathrm{f}'(x)$.

5E. $\mathrm{f}(x) = 6\cos 5x$. Find $\mathrm{f}'(x)$.

5P (a). $\mathrm{f}(x) = 2\sin 6x$. Find $\mathrm{f}'(x)$.

5P (b). $\mathrm{f}(x) = \frac{2}{3}\tan\left(\frac{1}{4}x\right)$. Find $\mathrm{f}'(x)$.

5E. $f(x) = \ln(3x + 5)$. Find $f'(x)$.

5P (a). $f(x) = \ln(2x - 7)$. Find $f'(x)$.

5P (b). $f(x) = 2\ln(3x^2 + 8x)$. Find $f'(x)$.

DIFFERENTIATING RECIPROCAL TRIGONOMETRIC FUNCTIONS | PROOFS

Using the chain rule, prove that $y = \sec x$ differentiates to $\frac{dy}{dx} = \sec x \tan x$

Using the chain rule, prove that $y = \operatorname{cosec} x$ differentiates to $\frac{dy}{dx} = -\operatorname{cosec} x \cot x$

Using the chain rule, prove that $y = \cot x$ differentiates to $\frac{dy}{dx} = -\operatorname{cosec}^2 x$