

$$y = \ln x \rightarrow \ln(x - 3)$$

$$\rightarrow \ln\left(\frac{1}{2}x - 3\right)$$



$$(a) f(x) \in (0, \infty)$$

$$(b) g(x) \in (-\infty, -1] \cup [1, \infty)$$



$$(a) x \geq 0, f(x) \geq 0$$

$$(b) x > 0, g(x) \in \mathbb{R}$$

$$(c) x \neq 2, h(x) \neq 0$$



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(1) Reflection in the x-axis

(2) Translation  $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$

or

(1) Translation  $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$

(2) Reflection in the x-axis



$$y = \ln x \rightarrow \ln\left(\frac{1}{2}x\right)$$

$$\rightarrow \ln\left(\frac{1}{2}x - \frac{3}{2}\right)$$



(1) Translation  $\begin{pmatrix} \frac{\pi}{3} \\ 0 \end{pmatrix}$

(2) Stretch, s.f.  $\frac{1}{2}$ , in x-direction  
or

(1) Stretch, s.f.  $\frac{1}{2}$ , in x-direction

(2) Translation  $\begin{pmatrix} \frac{\pi}{6} \\ 0 \end{pmatrix}$



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(a) No (not one-one)

(b) No (not one-one)

(b) Yes



$$\frac{2}{x+1} + \frac{5}{x-2} - \frac{4}{(x-2)^2}$$



(1) Stretch, s.f. 2, in y-direction

(2) Translation  $\begin{pmatrix} 0 \\ 5 \end{pmatrix}$

or

(1) Translation  $\begin{pmatrix} 0 \\ \frac{5}{2} \end{pmatrix}$



















(2) Stretch, s.f. 2, in y-direction



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<p> Domain and range of: <span style="border: 1px solid black; padding: 2px;">Technique</span></p> <p>(a) <math>f(x) = \sqrt{x}</math></p> <p>(b) <math>g(x) = \ln x</math></p> <p>(c) <math>h(x) = \frac{1}{x-2}</math></p> <p> 30s <span style="float: right;">A2-B1</span></p>	<p> Range of <span style="border: 1px solid black; padding: 2px;">Technique</span></p> <p>(a) <math>f(x) = e^x</math></p> <p>(b) <math>g(x) = \sec x</math></p> <p>using interval notation.</p> <p> 20s <span style="float: right;">A2-B2</span></p>	<p> <span style="border: 1px solid black; padding: 2px;">Technique</span></p> <p>Translation <math>\begin{pmatrix} 3 \\ 0 \end{pmatrix}</math>, followed by stretch, s.f. 2 in x-direction</p> <p><math>y = \ln x \rightarrow \dots</math></p> <p> 30s <span style="float: right;">A2-B3</span></p>
<p> Describe two different sequences of transformations: <span style="border: 1px solid black; padding: 2px;">Technique</span></p> <p><math>y = \sin x \rightarrow \sin\left(2x - \frac{\pi}{3}\right)</math></p> <p> 30s <span style="float: right;">A2-B4</span></p>	<p> Stretch, s.f. 2 in x-direction followed by translation <math>\begin{pmatrix} 3 \\ 0 \end{pmatrix}</math></p> <p><math>y = \ln x \rightarrow \dots</math></p> <p> 30s <span style="float: right;">A2-B5</span></p>	<p> Describe two different sequences of transformations: <span style="border: 1px solid black; padding: 2px;">Technique</span></p> <p><math>y = \cos x \rightarrow y = -\cos x + 2</math></p> <p> 30s <span style="float: right;">A2-B6</span></p>
<p> Describe two different sequences of transformations: <span style="border: 1px solid black; padding: 2px;">Technique</span></p> <p><math>y = e^x \rightarrow y = 2e^x + 5</math></p> <p> 30s <span style="float: right;">A2-B7</span></p>	<p> Write the following in terms of partial fractions: <span style="border: 1px solid black; padding: 2px;">Technique</span></p> $\frac{7x^2 - 17x - 6}{(x+1)(x-2)^2}$ <p> 2m 30s <span style="float: right;">A2-B8</span></p>	<p> Determine which of the following have an inverse: <span style="border: 1px solid black; padding: 2px;">Technique</span></p> <p>(a) <math>y = \cos x, 0^\circ &lt; x &lt; 360^\circ</math></p> <p>(b) <math>y = x^2, x \in \mathbb{R}</math></p> <p>(c) <math>y = e^x, x \in \mathbb{R}</math></p> <p> 30s <span style="float: right;">A2-B9</span></p>

$$\sin x \equiv \sin(x \pm 2\pi)$$

$$\cos x \equiv \cos(x \pm 2\pi)$$

$$\tan x \equiv \tan(x \pm \pi)$$

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$$\sin x = \sin(\pi - x)$$

$$\cos x = \cos(2\pi - x)$$

$$\text{(alternative: } \cos x = \cos(-x)\text{)}$$

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	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sin	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0

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$$\text{Note: } \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 2x \equiv \cos^2 x - \sin^2 x$$

$$\equiv 2 \cos^2 x - 1$$

$$\equiv 1 - 2 \sin^2 x$$

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$$\sin 2x \equiv 2 \sin x \cos x$$

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As  $x \rightarrow 0$ ,

$$\sin x \approx x$$

$$\cos x \approx 1 - \frac{1}{2}x^2$$

$$\tan x \approx x$$

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Deg	30°	45°	60°	90°	180°	360°
Rad	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$\pi$	$2\pi$

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$$\sec x \equiv \frac{1}{\cos x}, \quad \operatorname{cosec} x \equiv \frac{1}{\sin x}$$

$$\cot x \equiv \frac{\cos x}{\sin x} \equiv \frac{1}{\tan x}$$

Except when  $x = 0, \pi, \dots$

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$$\tan^2 x + 1 \equiv \sec^2 x$$

$$\cot^2 x + 1 \equiv \operatorname{cosec}^2 x$$

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Key Fact

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin					
cos					

30s

A2-E1



Key Fact

Symmetry properties for  
sin and cos (in radians)

15s

A2-E2



Key Fact

Periodic properties for sin,  
cos and tan (in radians)

20s

A2-E3



Key Fact

As  $x \rightarrow 0$ ,  
sin  $x \approx \dots$   
cos  $x \approx \dots$   
tan  $x \approx \dots$

10s

A2-E4



Key Fact

Double angle identity:  
sin  $2x \equiv \dots$

5s

A2-E5



Key Fact

Three double angle  
identities:  
cos  $2x \equiv \dots$

10s

A2-E6



Key Fact

2 identities linking  
sec  $x$ , cosec  $x$ , cot  $x$  and tan  $x$

10s

A2-E7



Key Fact

Fill in the gaps:

$$\sec x \equiv \frac{1}{\dots}, \quad \operatorname{cosec} x \equiv \frac{1}{\dots}$$

$$\cot x \equiv \frac{\dots}{\dots} \equiv \frac{1}{\dots}$$

10s

A2-E8

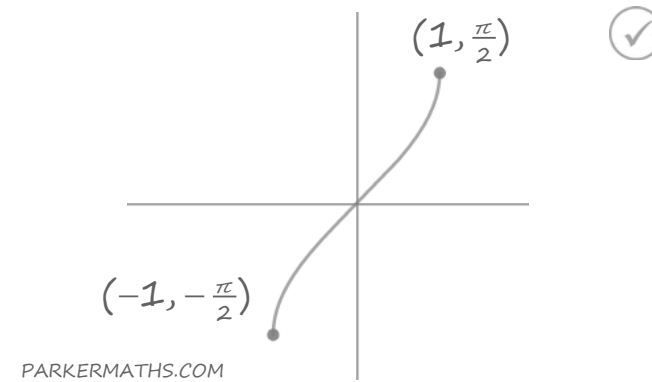
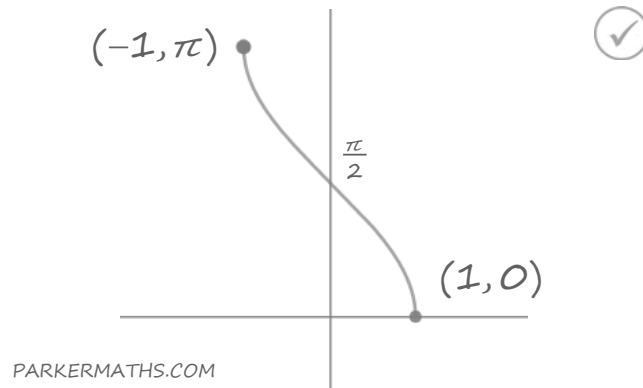
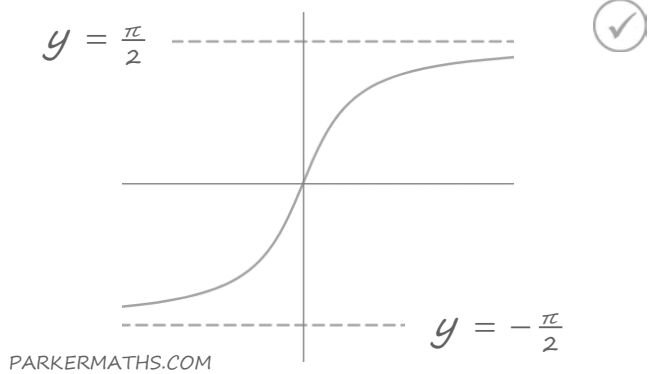


Key Fact

Deg	30°	45°	60°	90°	180°	360°
Rad						

25s

A2-E9



In radians:

$$\text{Arc length} = r\theta$$

$$\text{Sector area} = \frac{1}{2}r^2\theta$$

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$$y = a \sin bx$$

$$\text{Amplitude} = a$$

$$\text{Period} = \frac{2\pi}{b}$$

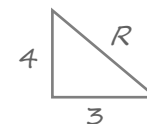
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$$3 \sin x - 4 \cos x \equiv R \sin(x - \alpha)$$

$$\equiv R \sin x \cos \alpha - R \cos x \sin \alpha$$

$$3 = R \cos \alpha \quad 4 = R \sin \alpha$$

$$\cos \alpha = \frac{3}{R} \quad \sin \alpha = \frac{4}{R}$$



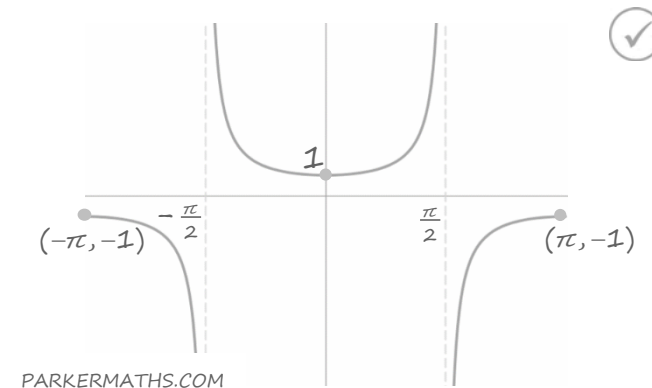
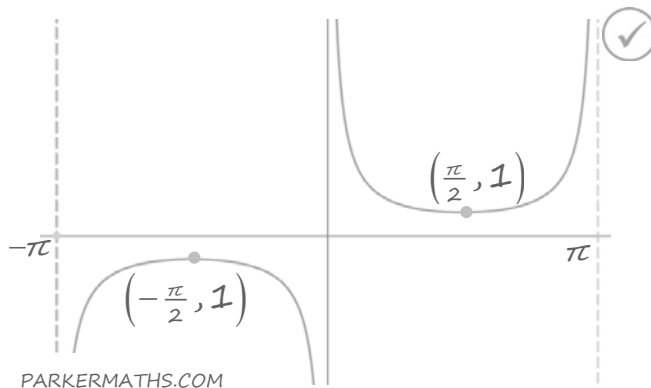
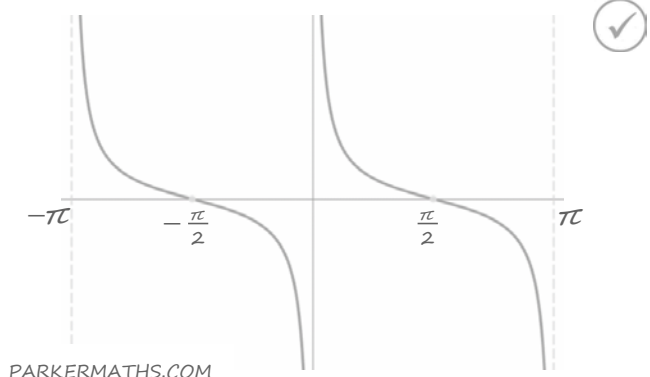
$$R^2 = 3^2 + 4^2$$



















$$\tan \alpha = \frac{4}{3}$$

$$R = 5$$

$$\alpha = 53.1^\circ$$

$$3 \sin x - 4 \cos x = 5 \sin(x - 53.1^\circ)$$

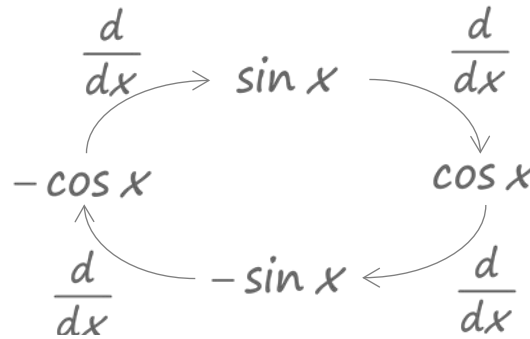


<p> <span style="float: right;">Key Fact</span></p> <p>Graph of <math>y = \sin^{-1} x</math> in radians showing intercepts and endpoints.</p> <p> 30s <span style="float: right;">A2-E10</span></p>	<p> <span style="float: right;">Key Fact</span></p> <p>Graph of <math>y = \cos^{-1} x</math> in radians showing intercepts and endpoints.</p> <p> 30s <span style="float: right;">A2-E11</span></p>	<p> <span style="float: right;">Key Fact</span></p> <p>Graph of <math>y = \tan^{-1} x</math> in radians showing intercepts and asymptotes.</p> <p> 30s <span style="float: right;">A2-E12</span></p>
<p> <span style="float: right;">Technique</span></p> <p>Given that</p> $3 \sin x - 4 \cos x = R \sin(x - \alpha)$ <p>Find the values of <math>R</math> and <math>\alpha</math></p> <p> 2m <span style="float: right;">A2-E13</span></p>	<p> <span style="float: right;">Key Fact</span></p> <p>In radians:</p> $y = a \sin bx$ <p>Amplitude = ...</p> <p>Period = ...</p> <p> 15s <span style="float: right;">A2-E14</span></p>	<p> <span style="float: right;">Key Fact</span></p> <p>In radians:</p> <p>Arc length = ...</p> <p>Sector area = ...</p> <p> 10s <span style="float: right;">A2-E15</span></p>
<p> <span style="float: right;">Key Fact</span></p> <p>Graph of <math>y = \sec x</math>, <math>-\pi \leq x \leq \pi</math></p> <p>Label asymptotes and stationary points.</p> <p> 60s <span style="float: right;">A2-E16</span></p>	<p> <span style="float: right;">Key Fact</span></p> <p>Graph of <math>y = \operatorname{cosec} x</math></p> $-\pi \leq x \leq \pi$ <p>(label asymptotes)</p> <p> 60s <span style="float: right;">A2-E17</span></p>	<p> <span style="float: right;">Key Fact</span></p> <p>Graph of <math>y = \cot x</math></p> $-\pi \leq x \leq \pi$ <p>(label asymptotes)</p> <p> 15s <span style="float: right;">A2-E18</span></p>

(a)  $3e^{3x+5}$

(b)  $(2x - 6)e^{x^2-6x}$

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(a)  $\frac{2}{2x+1}$

(b)  $\frac{2x-3}{x^2-3x}$

(c)  $\frac{1}{x}$



$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$



(a)  $-5 \sin(5x - 2)$

(b)  $\frac{1}{3} \cos\left(\frac{1}{3}x + 1\right)$



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$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

(a)  $2 \cos 2x$

(b)  $-3 \sin 3x$



(a)  $-30(3x - 1)^{-6}$

(b)  $2(4x + 5)^{\frac{1}{2}}$



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<p>?</p> <p>(a) <math>\frac{d}{dx} \ln(2x + 1) = \dots</math></p> <p>(b) <math>\frac{d}{dx} \ln(x^2 - 3x) = \dots</math></p> <p>(c) <math>\frac{d}{dx} \ln(5x) = \dots</math></p> <p>30s</p> <p>Key Fact</p> <p>A2-G1</p>	<p>?</p> <p><math>\frac{d}{dx}</math> <math>\sin x</math> <math>\frac{d}{dx}</math></p> <p>...</p> <p><math>\frac{d}{dx}</math> ... <math>\frac{d}{dx}</math></p> <p>15s</p> <p>Key Fact</p> <p>A2-G2</p>	<p>?</p> <p>(a) <math>\frac{d}{dx} e^{3x+5} = \dots</math></p> <p>(b) <math>\frac{d}{dx} e^{x^2-6x} = \dots</math></p> <p>10s</p> <p>Key Fact</p> <p>A2-G3</p>
<p>?</p> <p>(a) <math>\frac{d}{dx} \cos(5x - 2) = \dots</math></p> <p>(b) <math>\frac{d}{dx} \sin\left(\frac{1}{3}x + 1\right) = \dots</math></p> <p>10s</p> <p>Key Fact</p> <p>A2-G4</p>	<p>?</p> <p>Product rule for differentiation</p> <p>10s</p> <p>Key Fact</p> <p>A2-G5</p>	<p>?</p> <p>Chain rule for differentiation</p> <p>5s</p> <p>Key Fact</p> <p>A2-G6</p>
<p>?</p> <p>(a) <math>\frac{d}{dx} \frac{2}{(3x - 1)^5}</math></p> <p>(b) <math>\frac{d}{dx} \sqrt{4x + 5}</math></p> <p>20s</p> <p>Key Fact</p> <p>A2-G7</p>	<p>?</p> <p>(a) <math>\frac{d}{dx} \sin 2x = \dots</math></p> <p>(b) <math>\frac{d}{dx} \cos 3x = \dots</math></p> <p>10s</p> <p>Key Fact</p> <p>A2-G8</p>	<p>?</p> <p>Quotient rule for differentiation</p> <p>10s</p> <p>Key Fact</p> <p>A2-G9</p>



$$(a) \int \frac{3+2x}{x} dx = \int \left( \frac{3}{x} + 2 \right) dx \quad \checkmark$$

$$= 3 \ln |x| + 2x + c$$

$$(b) \int \frac{6}{3+2x} dx = 3 \ln |3+2x| + c$$

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$$(a) \int \dots dx = \int (2x+5)^{\frac{1}{2}} dx \quad \checkmark$$

$$= \frac{1}{3} (2x+5)^{\frac{3}{2}} + c$$

$$(b) \int \dots dx = 3 \int (4x+5)^{\frac{1}{2}} dx$$

$$= \frac{3}{2} (4x+5)^{\frac{1}{2}} + c$$

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$$(a) \frac{3}{2} \sin 2x + c \quad \checkmark$$

$$(b) -\frac{5}{3} \cos 3x + c$$

$$(c) -8 \cos\left(\frac{1}{4}x\right) + c$$

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$$\cos 6x = 1 - 2 \sin^2 3x \quad \checkmark$$

$$\sin^2 3x = \frac{1}{2} - \frac{1}{2} \cos 6x$$

$$\int \sin^2 3x = \int \left( \frac{1}{2} - \frac{1}{2} \cos 6x \right) dx$$

$$= \frac{1}{2}x - \frac{1}{12} \sin 6x + c$$

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$$\cos 2x = 2 \cos^2 x - 1 \quad \checkmark$$

$$\cos^2 x = \frac{1}{2} \cos 2x + \frac{1}{2}$$

$$\int \cos^2 x = \int \left( \frac{1}{2} \cos 2x + \frac{1}{2} \right) dx$$

$$= \frac{1}{4} \sin 2x + \frac{1}{2}x + c$$

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$$(a) \int \dots dx = 6 \int (3+2x)^{-5} dx \quad \checkmark$$

$$= -\frac{3}{4} (3+2x)^{-4} + c$$

$$(b) \int \dots dx = \int (9x^4 + 12x^3 + 4x^2) dx$$

$$= \frac{9}{5}x^5 + 3x^4 + \frac{4}{3}x^3 + c$$

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$$\int_{x_1}^{x_2} y dx = \int_{t_1}^{t_2} y \frac{dx}{dt} dt \quad \checkmark$$

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$$(a) \int \tan^2 x dx = \int (\sec^2 x - 1) dx \quad \checkmark$$

$$= \tan x - x + c$$

$$(b) \int \cot^2 x dx = \int (\operatorname{cosec}^2 x - 1) dx$$

$$= -\cot x - x + c$$

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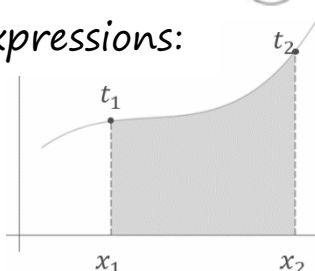
$$(a) \dots = \frac{1}{2}x \sin 2x - \frac{1}{2} \int \sin 2x dx \quad \checkmark$$

$$= \frac{1}{2}x \sin 2x + \frac{1}{4} \cos 2x + c$$

$$(b) \dots = \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^2 dx$$

$$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + c$$

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<p>?</p> <p>Technique</p> <p>(a) <math>\int 3 \cos 2x \, dx = \dots</math></p> <p>(b) <math>\int 5 \sin 3x \, dx = \dots</math></p> <p>(c) <math>\int 2 \sin\left(\frac{1}{4}x\right) \, dx = \dots</math></p> <p>25s A2-H1</p>	<p>?</p> <p>Technique</p> <p>(a) <math>\int \sqrt{2x+5} \, dx = \dots</math></p> <p>(b) <math>\int \frac{3}{\sqrt{4x+5}} \, dx = \dots</math></p> <p>30s A2-H2</p>	<p>?</p> <p>Technique</p> <p>(a) <math>\int \frac{3+2x}{x} \, dx = \dots</math></p> <p>(b) <math>\int \frac{6}{3+2x} \, dx = \dots</math></p> <p>30s A2-H3</p>
<p>?</p> <p>Technique</p> <p>(a) <math>\int \frac{6}{(3+2x)^5} \, dx = \dots</math></p> <p>(b) <math>\int (3x^2 + 2x)^2 \, dx = \dots</math></p> <p>30s A2-H4</p>	<p>?</p> <p>Technique</p> <p><math>\int \cos^2 x \, dx = \dots</math></p> <p>60s A2-H5</p>	<p>?</p> <p>Technique</p> <p><math>\int \sin^2 3x \, dx = \dots</math></p> <p>60s A2-H6</p>
<p>?</p> <p>Technique</p> <p>(a) <math>\int x \cos 2x \, dx = \dots</math></p> <p>(b) <math>\int x^2 \ln x \, dx = \dots</math></p> <p>2m A2-H7</p>	<p>?</p> <p>Technique</p> <p>(a) <math>\int \tan^2 x \, dx = \dots</math></p> <p>(b) <math>\int \cot^2 x \, dx = \dots</math></p> <p>Tip: Identities and formula booklet.</p> <p>60s A2-H8</p>	<p>?</p> <p>Key Fact</p> <p>Formula for integrating parametric expressions:</p> <p><math>\int_{x_1}^{x_2} y \, dx = \dots</math></p>  <p>15s A2-H9</p>