

$$1 \quad \begin{array}{lll} \mathbf{a} = \log_{10} a + \log_{10} b & \mathbf{b} = \log_{10} a + \log_{10} b^7 & \mathbf{c} = \log_{10} a^3 - \log_{10} b \\ & = \log_{10} a + 7 \log_{10} b & = 3 \log_{10} a - \log_{10} b \\ \mathbf{d} = \log_{10} a + \log_{10} b^{\frac{1}{2}} & & = \log_{10} a + \frac{1}{2} \log_{10} b \\ \mathbf{e} = 2 \log_{10} ab & \mathbf{f} = -\log_{10} ab & \mathbf{g} = \log_{10} a^{\frac{3}{2}} + \log_{10} b^{\frac{5}{2}} \\ = 2 \log_{10} a + 2 \log_{10} b & = -\log_{10} a - \log_{10} b & = \frac{3}{2} \log_{10} a + \frac{5}{2} \log_{10} b \\ \mathbf{h} = 3(\log_{10} a^2 - \log_{10} b^{\frac{1}{3}}) & & = 6 \log_{10} a - \log_{10} b \end{array}$$

$$2 \quad \begin{array}{lll} \mathbf{a} = \log_q 8^2 & \mathbf{b} = \log_q 8^{\frac{1}{3}} & \mathbf{c} = \log_q 16 - \log_q q \\ = 2y & = \frac{1}{3}y & = \log_q 8^{\frac{4}{3}} - 1 \\ & & = \frac{4}{3}y - 1 \\ \mathbf{d} = \log_q 4 + \log_q q^3 & & = \log_q 8^{\frac{2}{3}} + 3 \\ & & = \frac{2}{3}y + 3 \end{array}$$

$$3 \quad \begin{array}{lll} \mathbf{a} = \lg(2 \times 3^2) & \mathbf{b} = \lg(2^5 \times 3) & \mathbf{c} = \lg 9 - \lg 16 \\ = \lg 2 + 2 \lg 3 & = 5 \lg 2 + \lg 3 & = \lg 3^2 - \lg 2^4 \\ = a + 2b & = 5a + b & = 2 \lg 3 - 4 \lg 2 \\ & & = 2b - 4a \\ \mathbf{d} = \lg(2 \times 3) - \lg 2^3 & & = \lg 2 + \lg 3 - 3 \lg 2 \\ & & = \lg 3 - 2 \lg 2 \\ & & = b - 2a \\ \mathbf{e} = \frac{1}{2} \lg 6 & \mathbf{f} = \frac{3}{2} \lg 2^4 + \frac{1}{2} \lg 3^4 & \mathbf{g} = 4 \lg 3 - 3(\lg 2 + \lg 3) \\ = \frac{1}{2}(\lg 2 + \lg 3) & = 6 \lg 2 + 2 \lg 3 & = \lg 3 - 3 \lg 2 \\ = \frac{1}{2}(a + b) & = 6a + 2b & = b - 3a \\ \mathbf{h} = \lg(6 \times 10) + \lg(2 \times 10) - 2 & & = \lg 6 + 1 + \lg 2 + 1 - 2 \\ & & = \lg 2 + \lg 3 + \lg 2 \\ & & = 2a + b \end{array}$$

$$4 \quad \begin{array}{lll} \mathbf{a} = \log_5 10 - \log_5 2 & \mathbf{b} = \log_{12} 16 + \log_{12} 9 & \mathbf{c} = \log_4 8 \\ = \log_5 5 & = \log_{12} 144 & = \log_4 4^{\frac{3}{2}} \\ = 1 & = 2 & = \frac{3}{2} \\ \mathbf{d} = \frac{\log_7 3^4}{\log_7 3} & \mathbf{e} = \log_{27} \frac{12^3}{72^2} & \mathbf{f} = \frac{\log_{11} 5^2}{-\log_{11} 5} \\ = \frac{4 \log_7 3}{\log_7 3} & = \log_{27} \frac{12 \times 12 \times 12}{6 \times 12 \times 6 \times 12} & = \frac{2 \log_{11} 5}{-\log_{11} 5} \\ = 4 & = \log_{27} \frac{1}{3} = -\frac{1}{3} & = -2 \end{array}$$

$$5 \quad \begin{array}{lll} \mathbf{a} \quad x = 3^{1.8} & \mathbf{b} \quad x = 5^{-0.3} & \mathbf{c} \quad x - 3 = 8^{2.1} \\ x = 7.22 & x = 0.617 & x = 3 + 8^{2.1} \\ & & x = 81.8 \\ \mathbf{d} \quad \frac{1}{2}x + 1 = 4^{3.2} & \mathbf{e} \quad \log_2 3y = 5.3 & \mathbf{f} \quad \log_6(1 - 5t) = -0.6 \\ x = 2(4^{3.2} - 1) & 3y = 2^{5.3} & 1 - 5t = 6^{-0.6} \\ x = 167 & y = \frac{1}{3} \times 2^{5.3} & t = \frac{1}{5}(1 - 6^{-0.6}) \\ & y = 13.1 & t = 0.132 \end{array}$$

$$6 \quad \begin{array}{lll} \mathbf{a} = \log_2 x^5 & \mathbf{b} = \log_2(x^2 + 4x) & \mathbf{c} = \log_2 x^2 + \log_2 x \\ & & = \log_2 x^3 \\ \mathbf{d} = \log_2(x - 2)^3 - \log_2 x^4 & \mathbf{e} = \log_2 \frac{x^2 - 1}{x + 1} & \mathbf{f} = \log_2 x - 2 \log_2 x + \frac{2}{3} \log_2 x \\ = \log_2 \frac{(x - 2)^3}{x^4} & = \log_2 \frac{(x + 1)(x - 1)}{x + 1} & = -\frac{1}{3} \log_2 x \\ & = \log_2(x - 1) & = \log_2 x^{-\frac{1}{3}} \end{array}$$

7 a  $\log_3 5x = \log_3 (2x + 3)$

$$5x = 2x + 3$$

$$x = 1$$

c  $\log_4 \frac{x}{x-1} = \log_4 3 + \log_4 2 = \log_4 6$

$$\frac{x}{x-1} = 6$$

$$x = 6x - 6$$

$$x = \frac{6}{5}$$

e  $\log_6 x^2 = \log_6 5(2x - 5)$

$$x^2 = 5(2x - 5)$$

$$x^2 - 10x + 25 = 0$$

$$(x - 5)^2 = 0$$

$$x = 5$$

8 a  $\log_x y = 2 \Rightarrow y = x^2$

sub.  $x^3 = 27$   
 $x = 3$   
 $\therefore x = 3, y = 9$

c  $\log_y 32 = -\frac{5}{2} \Rightarrow y^{-\frac{5}{2}} = 32$   
 $\Rightarrow y = 32^{-\frac{2}{5}} = \frac{1}{4}$

sub.  $\log_2 x = 3 - 2 \log_2 \frac{1}{4}$   
 $\log_2 x = 3 - (-4) = 7$   
 $x = 2^7 = 128$   
 $\therefore x = 128, y = \frac{1}{4}$

e  $\log_a x + \log_a 3 = \frac{1}{2} \log_a y \Rightarrow 3x = y^{\frac{1}{2}}$   
 $\Rightarrow y = 9x^2$

sub.  $3x + 9x^2 = 20$   
 $9x^2 + 3x - 20 = 0$   
 $(3x + 5)(3x - 4) = 0$   
 for real  $\log_a x, x > 0 \therefore x = \frac{4}{3}$   
 $\therefore x = \frac{4}{3}, y = 16$

b  $\log_9 10x = \frac{3}{2}$

$$10x = 9^{\frac{3}{2}} = 27$$

$$x = 2.7$$

d  $\log_5 \frac{5x}{x+2} = \log_5 \frac{x+6}{x}$

$$\frac{5x}{x+2} = \frac{x+6}{x}$$

$$5x^2 = (x+2)(x+6) = x^2 + 8x + 12$$

$$x^2 - 2x - 3 = 0$$

$$(x+1)(x-3) = 0$$

$$x = -1, 3$$

$$\log_5 x \text{ not real for } x = -1 \therefore x = 3$$

f  $\log_7 4x - \log_7 \frac{1}{x-6} = 1$

$$\log_7 4x(x-6) = 1$$

$$4x(x-6) = 7$$

$$4x^2 - 24x - 7 = 0$$

$$x = \frac{24 \pm \sqrt{576 + 112}}{8} = 3 \pm \frac{1}{2}\sqrt{43}$$

$$\log_7 4x \text{ not real for } x = 3 - \frac{1}{2}\sqrt{43}$$

$$\therefore x = 3 + \frac{1}{2}\sqrt{43} \quad [= 6.28 \text{ (3sf)}]$$

b  $\log_5 x - 2 \log_5 y = \log_5 2 \Rightarrow \frac{x}{y^2} = 2$   
 $\Rightarrow x = 2y^2$

sub.  $3y^2 = 12$   
 $y^2 = 4$   
 for real  $\log_5 y, y > 0 \therefore y = 2$   
 $\therefore x = 8, y = 2$

d  $\log_y x = \frac{3}{2} \Rightarrow y^{\frac{3}{2}} = x$   
 $\Rightarrow y^{\frac{1}{2}} = x^{\frac{1}{3}}$

sub.  $4x^{\frac{1}{3}} = 20$   
 $x^{\frac{1}{3}} = 5$   
 $x = 5^3 = 125$   
 $\therefore x = 125, y = 25$

f  $\log_{10} y + 2 \log_{10} x = 3 \Rightarrow x^2 y = 10^3$

$$\log_2 y - \log_2 x = 3 \Rightarrow \frac{y}{x} = 2^3$$

$$\Rightarrow y = 8x$$

sub.  $8x^3 = 1000$   
 $x^3 = 125$   
 $x = 5$   
 $\therefore x = 5, y = 40$